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REPORT R-1653

A TRIFILAR PENDULUM

FOR

THE DETERMINATION OF MOMENTS OF INERTIA

BY

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and

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REPORT R-1653

A TRIFILAR PENDULUM

FOR

THE DETERMINATION OF MOMENTS OF INERTIA

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ABSTRACT

A unique trifilar pendulum was constructed at Frankford Arsenal which proved to be an accurate tool for the determination of moments of inertia of non-homogeneous objects.

The pendulum was used with a calibration curve. The curve was generated by plotting a quantity calculated from the weight and the period squared of a number of homogeneous solids of revolution of increasing size as ordinates, against the calculated (using handbook formulae) moments of inertia of these solids as abscissae.

The moment of inertia of an unknown was obtained by weighing the object, establishing its period on the pendulum, locating a quantity calculated from its period squared and its weight as an ordinate on the calibration curve, and finding the moment of inertia as the corresponding abscissa.

There are specialized techniques used in operating the pendulum and some unique features in its design for which patents have been applied. The need for such unique features is established, and error analysis shows this trifilar pendulum accurate to 0.5%.

It is concluded that the trifilar pendulum, if properly constructed, has certain advantages over the torsion rod pendulum with no more disadvantages.

NOMENCLATURE

$\vec{i}, \vec{j}, \vec{k}$	Rectangular coordinate system unit vectors.
\vec{R}	Vector from axis of rotation of Trifilar Pendulum to one wire of trifilar pendulum and perpendicular to axis of rotation.
R	Magnitude of \vec{R} .
L	Length of suspending wires.
T	Period of oscillation of Trifilar Pendulum.
w	Weight.
α (alpha)	Rotational displacement of weight suspended on trifilar pendulum.
η (eta)	Reference angle orienting \vec{R} with respect to rectangular coordinate system.
β (beta)	Complement of η ; also, angle between incident and reflected light beams at moving mirror on pendulum.
θ (theta)	Angle between pendulum wire and vertical under displacement.
γ (gamma)	Time constant of attenuation of motion of trifilar pendulum.
ϕ (phi)	Phase angle associated with periodic motion of trifilar pendulum.
τ (tau)	Torque; also, probable error.
ω (omega)	Frequency of oscillation of trifilar pendulum; by definition,

$$\omega = \frac{2\pi}{T}$$

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INTRODUCTION

In July of 1957, a project was undertaken to build a pendulum capable of determining the moment of inertia of a device of approximately 4 lbs in weight, having an odd configuration. Since a time limitation was given to produce the pendulum at the lowest possible cost, it was decided to build a trifilar pendulum, from scrap material available at Frankford Arsenal.

The resulting pendulum has a configuration of structural elements, consisting of tubing, angles and plates of a size available at the time of construction. The mirrors used were manufactured from laboratory coverglasses which were silvered by our optical shop. The photocell and amplifier were part of surplus equipment. The only purchased parts were timers and a counter.

After the pendulum had been built and had proven its worth by the relative accuracy of its results, a patent application was filed based on a number of its unique characteristics.

DISCUSSION

In dynamics the moment of inertia of a body is to rotary motion as its mass is to linear motion. Engineers, therefore, should, in any investigation, concern themselves with the values used for moments of inertia as they do for values of mass.

In general, an investigator may calculate the value of a moment of inertia or find it by experiment. For irregular or nonhomogeneous solids, calculation is usually impractical. This report describes a device for determining moments of inertia of solids unamenable to calculation, and the method of its operation. The device is called a trifilar pendulum.

A simple pendulum consists of an oscillating weight hung from a single wire. The weight, or bob, is free to swing in a plane (or in an elliptical path, which is just two plane paths superimposed) or rotate about an axis at a constant rate.

A torsion rod pendulum (used in empirical methods of finding moments of inertia) also has a single wire, but the wire is thick enough to exert a measurable restoring torque on the bob when the wire is twisted. It should be remembered that moments of inertia (unless calculated) may only be found experimentally by imparting an angular acceleration to the object under investigation. Physically, there cannot be a scale or balance or any static test for finding moments of inertia.

In the trifilar (three-wire) pendulum, the wire size used is as thin as the elastic limit will allow. Thin wires, of the size used, have a torsional moment small enough to ignore. The wires are hung so that when the bob has been rotated and set free, a restoring torque twists it back toward the equilibrium position, and periodic motion sets in (fig. 1). The weight and moment of inertia of the bob and some dimensions of the pendulum fix the period.

THEORY

1. Appendix I contains the derivation of the formula for moment of inertia from the equation of motion of the trifilar pendulum:*

$$I = \frac{WR^2T^2}{4\pi^2L}$$

*G. W. Hughes, Sandia Corp., The Trifilar Pendulum and Its Application to the Experimental Determination of Moments of Inertia (ASME 57-SA-51, 1957) p. 42.

Where

I = moment of inertia of the bob (test object plus hardware)

W = weight of the bob

R = perpendicular distance from the axis of the bob (about which it rotates) to the center line of any one of the wires (fig. 1)

T = period for one full cycle

L = length of wire from the supporting frame to the bob.

METHOD

The technique used in determination of moments of inertia of unknown nonhomogeneous solids by use of the trifilar pendulum involves the use of a calibration curve. This curve is obtained by (1) calculating the values of the moments of inertia of standards; (2) weighing each standard (W_s); (3) weighing the holding fixture (W_o); mounting each standard on the holding fixture; (5) imparting an oscillation to the assembly; (6) determining the period (T_{s+o}); (7) imparting an oscillation to the holding fixture without a standard; (8) determining its period (T_o); (9) calculating the quantity $(W_s + W_o)(T_{s+o})^2 - W_o T_o^2$; and, (10) plotting this value as ordinate against the corresponding calculated moment of inertia as abscissa.

The moment of inertia of an unknown is found by (1) weighing it (W_x); (2) mounting it on the holding fixture; (3) imparting an oscillation to the assembly; (4) determining the period (T_{x+o}); (5) calculating the quantity $(W_x + W_o)(T_{x+o})^2 - W_o T_o^2$; (5) finding the point on the calibration curve whose ordinate is this quantity; and (6) reading the abscissa of this point as the desired moment of inertia. This value is the moment of inertia of the unknown.

If the unknown could be oscillated without using the holding fixture, one would obtain the period of oscillation of just the unknown (T_x). A holding fixture is necessary, however, and, in general, $T_x \neq T_{x+o}$. Thus $W_x T_x^2$, which is the factor in the formula for the moment of inertia of just the unknown (see App. I), must be found indirectly. Let I_o stand for the moment of inertia of just the holding fixture, I_x that of the unknown, and I_{x+o} that of the combination. Adding moments of inertia

$$I_{x+o} = I_x + I_o$$

Using the trifilar pendulum formula (see App. I) and cancelling constants,

$$(W_x + W_o) (T_{x+o})^2 = W_x T_x^2 + W_o T_o^2$$

so

$$W_x T_x^2 = (W_x + W_o) (T_{x+o})^2 - W_o T_o^2$$

The amount on the right is the WT^2 used as ordinate of the calibration curve. I_x is the abscissa, so the unknown moment of inertia can be read directly from the graph.

The standards used to provide points for the calibration curve were homogeneous cylinders of steel or aluminum, accurately weighed and measured. Theoretically true moments of inertia (axial and transverse) were calculated using handbook formulae.

The standards are divided into two families; one set had a constant height with increasing radius while the other had constant radius and increasing height.

Each of these standards had its period timed for both axial and transverse axes. Each standard, then, gave two points for the calibration curve. There were six standards in each of the two families, so a total of twenty-four points was collected. The weight and period of the hardware alone gives another point which is, of course, the origin of the coordinates.

CONSTRUCTION AND METHOD OF OPERATION

Three pieces of aluminum tube rise from the floor and connect at the top and middle to crosswise pieces of angle iron, forming the frame. A triangular yoke of flat pieces of steel welded together is

bolted to the midpoints of the top angle irons (figs. 2 & 3). It was originally planned to have the uprights join right to the yoke (fig. 4). Over the yoke and bolted to it lies a circular steel plate called the "top support plate." Three sets of three tapped holes, forming the vertices of concentric equilateral triangles, go through the plate.

Levelling and locking bolts screw into one set of holes. A level is placed across the heads of each pair of bolts and one is raised or lowered until level with the other. When all three are level, locking nuts are tightened. The bolts have thin axial holes in which the wires are inserted and held by a setscrew.

There are three sets of holes in the top support plate, so there are three possible values of R (see Appendix I). In practice, the center-most holes were usually used. The formula for I shows that the period varies inversely with R (everything else constant). Hence, the smaller the R , the longer the period, and the smaller the percentage error in T . Other efforts in three-wire pendulum design can be criticized on this account. In particular, the dimensions of the pendula constructed by G. W. Hughes* lead to periods in the neighborhood of a fraction of a second.

The lower support plate has thin vertical wire holes and setscrews near the edge. It is levelled the same way as the upper support plate, except that one setscrew must be loosened before raising or lowering that side of the plate, then tightened again.

A holding rig hangs from the bottom support plate by a hex bolt (figs. 5 & 6). This rig is used for finding transverse moments of inertia, and is little more than a rectangular aluminum frame. Atop its lower member sits a piece of angle on which rides the test object (fig. 11). Holding the angle in place and extending below the frame is a holding and positioning bolt. The bottom of the holding and positioning bolt has a tapered bore, which mates with a conical steel shaft threaded into the intersection of a pair of center bearing arms. These arms move in collars attached to legs of the frame. The shaft can move in all directions. It is centered (before the wires are hung) by means of a plumb bob hung from the center of the top support plate.

*G. W. Hughes, Sandia Corp., The Trifilar Pendulum and Its Application to the Experimental Determination of Moments of Inertia (ASME 57-SA-51, 1957) p. 42.

The center bearing arm is needed because the three-wire pendulum must be kept from swinging like a simple pendulum. As shown in App. II, when there is both rotational and simple pendular motion, the horizontal and vertical displacements differ from wire to wire.

This combination of motions has many undesirable results.

1. Since the angle between a wire and vertical $\theta = \tan \theta \doteq \frac{S}{L}$ (the displacement of the bottom end of the wire divided by the wire's length), the three wires are at different angles to the vertical, upsetting one of the assumptions of the derivation.
2. The axis of rotation of the bob and the direction of the restoring torque are no longer vertical or parallel. The axis of rotation moves in one plane while the torque is not held in any plane. At any instant, the test object is spinning about an unknown axis and accelerating angularly about another. (In referring to or determining the moment of inertia, a reference axis must be specified.)
3. These torsional and simple pendular motions also have separate periods. It follows that results under these conditions become meaningless.
4. It has been found that the rotational motion damps rapidly when there is also a simple pendular motion.
5. It will be noted further in the report that a simple pendular swing would make it impossible to get results by the timing method used.

It is concluded that:

1. The operator should have some tool to help him start the pendulum without jolting it.
2. A fixed pin is needed under the bob to keep it from swinging like a simple pendulum, so that results are obtained and are meaningful.

These conclusions, if not the figuring that leads to them, are valid for all types of torsion pendula.

The shaft in the center bearing arm referred to above is raised by means of its threaded base as far as it can without bearing the suspended weight. At most, the shaft and matching depression touch at a point (fig. 6 inset). Variations in temperature and variations in the weight of the test object affect the length of the wires so that the shaft must be raised or lowered to maintain single point bearing. Needless to say, if the hole in the middle of the top support plate is badly centered, the shaft will move the bob sideways. This sideways shift will have much the same result as a simple pendular swing.

The frame has a pair of pins on its side members. Before being bolted to the bottom support plate, the frame, with test object, hangs by these pins on a special cradle. (Fig. 13.) One of the cradle legs has a jack for levelling. The test object is shifted until the whole works hangs vertical. When the frame hangs vertical, i. e., when the test object is balanced, an engraved line on one side of the frame lines up with a wire by one of the cradle legs. Thus, when the frame and test object are put on the pendulum, the axis of rotation goes through the bob's center of mass.

Usually, it is the moment of inertia about an axis through the center of mass that is desired. But the three-wire pendulum has the virtue of allowing some range of arbitrary axes. True, a simple relation exists between a moment of inertia through the center of mass and others parallel to it. But if the location of the axis for the wanted moment of inertia is known, calculation is unnecessary using this axis in the experiment. It is easy to show that the bob will not tilt until the distance of the center of mass from the axis of rotation exceeds

$$\frac{R \sqrt{3}}{6}$$

In this regard, the three-wire pendulum is better than the torsion rod pendulum, which will tilt under any imbalanced load.

Objects having axial mounting holes, and whose axial moments of inertia are to be found, are hung from an expansion plug bolted to the bottom support plate in place of the frame (figs. 7 & 8). Solid objects whose axial moment of inertia are to be determined sit atop the bottom support plate.

A tool has been made for measuring the wire length, such as, for an absolute determination of I . This tool works as follows: a pair of rods slide in and out of a collar and can be locked in place by the collar (fig. 9). The free ends of the rods have disks on them. A spacer is put on the bottom support plate and one of the disks of the extension rod set on it. The rod is extended until the upper end butts against the locking bolts, then locked. Vernier calipers then measure the lengths of the spacer and rod.

A keyed sleeve which slides freely up and down is used for tremor-free starting (figs. 10, 11, & 13). Resting around the guide plug, it slips up around the holding & positioning bolt and keys into a groove. After keying, sleeve and bolt are turned through a desired angle and released. The sleeve drops, letting the pendulum rotate undisturbed. Fixed to the centering arm is a circular plate with radial lines. A positioning arm on the keyed sleeve can be lined up with one of these lines so that an amplitude can be chosen. An amplitude of about 7° has been found best.

A polyethline sheet hangs around the pendulum to cut out drafts. The pendulum was located in an air conditioned room so that the ambient temperature variation was $\pm 1^\circ \text{C}$.

An electronic setup times the period, (fig. 12). A light beam bounces off a narrow mirror on one face of the hex bolt or on the edge of the bottom support plate. The reflected beam then carves out an arc with the same frequency as the pendulum. When the pendulum is at its equilibrium position, the reflected beam shines into a photocell. (The light source and photocell are attached to two of the frame up-rights.) Light hits the photocell twice every period and a counter records the number of hits. The counter is wired to two electronic timers. One of them times the first ten full cycles, then stops to let the operator record this time while the other one starts timing. The operator resets the first timer when he is done recording, then turns to the second timer and repeats the operation. This goes on until ten readings have been obtained from each timer, or until the beam moves so slow that the photocell fires twice while the beam shines on it. (Figs. 12 & 13.)

This way of timing the period would obviously get no satisfactory results if the pendulum swung as a simple pendulum. The reflected light beam would be very erratic, missing the photocell or striking it irregularly.

ERROR ANALYSIS

Sources of error are divided into two groups; those caused by the three-wire pendulum design, and those caused by the setup used for timing.

Sources of error caused by the design are: (1) changes in R; (2) changes in L; (3) error in weighing.

Since

$$I = \frac{WR^2T^2}{4\pi^2L}$$

$$\frac{dI}{dR} = \frac{2WRT^2}{4\pi^2L}$$

$$\left| \frac{dI_R}{I} \right| = \left| \frac{2WRT^2}{4\pi^2L} \cdot \frac{4\pi^2L}{WR^2T^2} \right| \left| dR \right| = 2 \left| \frac{dR}{R} \right|$$

The subscript on dI stands for the source of error.

R varies with the ambient temperature, so that $dR = KR\Delta T$, K the coefficient of expansion, ΔT the change in temperature, say 2°C . For aluminum $K = 2.57 \times 10^{-5}/^\circ\text{C}$

so

$$\begin{aligned} 2 \left| \frac{dR}{R} \right| &= 2K\Delta T \\ &= 1.03 \times 10^{-4} \\ &= 0.01\% \end{aligned}$$

The weight of the unknown will stretch the wires. The modulus of elasticity $E = WL/A_e$, where w is the weight of the bob, L the wire length, A the cross section area of the wire, and e the elongation of the wire

$$\frac{dI_L}{I} = \left| \frac{dL}{L} \right| = \frac{e}{L}$$

$$= \frac{W}{EA}$$

The weigh. supported by one wire will stay less than 100 lb. The cross section area of the wire is about $8.1 \times 10^{-4} \text{ in}^2$, and the modulus of elasticity of steel is about 2.8×10^7 . Hence,

$$\frac{dI_L}{I} = \frac{100}{2.8 \times 10^7 \times 8.1 \times 10^{-4}}$$

$$= 4.4 \times 10^{-3}$$

$$= 0.44\%$$

Even though a calibration curve is used, this source of error must be counted. $WT^2/I = C$ is considered constant for all possible test objects. If the wires stretch for a certain test object, then

$$dI = I' - I = WT^2 C - I$$

$$= WT^2 \left(C - \frac{I}{WT^2} \right)$$

Where I' is the experimentally determined value of moment of inertia, I is the true value, C is the slope of the calibration curve, W = the weight of the test object and T is its period.

$$dI = I' - I = WT^2 \frac{R^2}{4\pi^2} \left(\frac{1}{L} - \frac{1}{L'} \right)$$

where L' is the new length of the wire, L the old.

$$= \frac{WT^2 R^2}{4\pi^2} \left(\frac{e}{LL'} \right)$$

$$= \frac{WT^2 R^2}{4\pi^2 L} \left(\frac{e}{L'} \right) = I \left(\frac{e}{L'} \right)$$

hence

$$\frac{dI}{I} = \frac{e}{L}$$

Differentiating the formula for I with respect to weight,

$$\frac{dIw}{I} = \frac{dw}{w}$$

Standards and unknowns are weighed on a lever arm balance and are known to better than one part in 10^3 so $dI_w/I = 0.1\%$.

The effect of air drag on the period is negligible.

The smallest measurable interval is a millisecond. The shortest periods timed are on the order of a few seconds. A timing run has ten cycles, and both the starting and ending times can be off by as much as a millisecond. Hence,

$$\frac{dI_{T_1}}{I} = \frac{2(dT_1)}{T} = \frac{2 \times 2 \times 10^{-3}}{0.04} = 4 \times 10^{-4}$$

$$= 0.04\%$$

The number subscript is used because there are other sources of error in T .

If the pendulum is not at the equilibrium point when the photocell goes off, the measured period at this point will be a little longer than the true period. Suppose the photocell fires when the pendulum reaches angle $\alpha_1 = \alpha_0 e^{-\gamma t_1} \cos wt_1$.

Where α_0 is the amplitude, $\cos wt_1$ is the phase factor, and $e^{-\gamma t_1}$ the damping factor. After one full period, the time is $t_1 + T$. (T the true period), but the photocell has not yet fired, because the amplitude has decreased since t_1 . It does not fire until a little later, for example $t_1 + T + dT$. The interval actually timed is $T + dT$, i. e., the true period plus a little bit.

The derivation of dT_2/T (subscript for identification) is given in Appendix III. Here is the result:

$$\frac{dT_2}{T} = \frac{1}{2\pi} \frac{\alpha_1}{\sqrt{(\alpha_0 e^{-\gamma t_1})^2 - \alpha_1^2}} \left[e^{\gamma T} - 1 \right]$$

assuming

$$\alpha_1 = 5^\circ \alpha_0 e^{-\gamma t_1} = 7^\circ$$

and

$$\gamma \doteq \left(\frac{1}{1000T} \right) \quad (e^{\gamma T} = 1 + \gamma T, \gamma t \text{ small})$$

We arrive at a worst possible error.

$$\begin{aligned} \left| \frac{dT_2}{T} \right| &= \frac{1}{6.28} \left[\frac{5}{\sqrt{49-25}} \right] \left[1 + \frac{1}{1000} - 1 \right] \\ &= 0.02\% \end{aligned}$$

hence

$$\left| \frac{dI_{T2}}{I} \right| = 0.04\%$$

Note

$$\left| \frac{dT_2}{T} \right| \rightarrow 0 \text{ as } \alpha_1 \rightarrow 0$$

and

$$\left| \frac{dT_2}{T} \right| \rightarrow \infty \text{ as } \alpha_1 \rightarrow \alpha_0 e^{-\gamma t_1}$$

(because of an increase in α_1 , a decrease in α_0 , increase in γ , or increase in t_1 .)

If the apparatus is set up properly, this source of error can be ignored. It was merited to show that the location of the photocell is not haphazard.

The mirror which reflects the light beam and the slit which collects it both have a finite width in the plane of motion. Because of this finite width, the pendulum bob can be anywhere within a small angular range when the photocell is triggered. It follows that the measured period may be off by a small amount.

The magnitude of this possible error is derived in Appendix IV. It turns out to be

$$\frac{dT_3}{T} = \frac{W_1 + W_2}{20\pi\alpha_0 D}$$

Where W_1 is the width of the mirror, W_2 the width of the slit before the photocell, α_0 is the pendulum's amplitude, and D the distance from the mirror to the photocell. The error is figured for a timing run of ten periods; if single periods are timed dT_3/T is ten times as great.

Assuming α_0 about $\frac{1}{3\pi}$ radians,

$$W_1 \doteq \left(\frac{1}{16}\right)''$$

$$W_2 = \left(\frac{1}{8}\right)''$$

and

$$D \doteq 15''$$

$$\left| \frac{dT_3}{T} \right| = \frac{1}{60} \left[\frac{\frac{3}{16}}{\frac{15}{15}} \right] = \frac{1}{4800} = 0.02\%$$

Hence

$$\frac{dI_{T_3}}{I} = 0.04\%.$$

Let us add the errors from all the sources mentioned:

$$\begin{aligned} \left| \frac{dI}{I} \right| &= \left| \frac{dI_R}{I} \right| + \left| \frac{dI_W}{I} \right| + \left| \frac{dI_L}{I} \right| + \left| \frac{dI_{T_1}}{I} \right| + \left| \frac{dI_{T_3}}{I} \right| \\ &= 2 \left| \frac{dR}{R} \right| + \left| \frac{dW}{W} \right| + \left| \frac{dL}{L} \right| + \left| \frac{2dT_1}{T} \right| + \left| \frac{2dT_3}{T} \right| \\ &= 0.01\% + 0.1\% + 0.44\% + 0.04\% + 0.04\% \\ &= 0.63\% \approx 0.6\% \end{aligned}$$

An actual calibration curve is a straight line (graph #1). If every point (WT^2, I) lay exactly on the curve, then $(WT^2/I)_i$ would be a constant for all i . It is not.

In effect, each set of numbers (I, WT^2) can be considered a separate determination of the theoretically constant value $\left(\frac{WT^2}{I} \right)$

(graph #2). Some values of WT^2/I are not as reliable as others. Some are known to four significant figures; some to three; some only to two. So take a weighted mean with the following table for determining weighting factors:

Significant Figures

Weighting Factor

4

1

3

.1

2

.01

The weighted mean

$$\bar{m} = \frac{\sum_{i=1}^n p_i m_i}{\sum_{i=1}^n p_i}$$

where

$$\bar{m} = \frac{\overline{WT^2}}{I}$$

$$m_i = \left(\frac{WT^2}{I} \right)_i$$

and p_i is the i th weighting factor.

There are twenty-four points with a total weighting of 15.81,
and $\bar{m} = 0.8505$.

The probable error of an observation with weight 1

$$\tau = \frac{0.6745}{\sqrt{n-1}} \left(\sum_{i=1}^n p_i x_i^2 \right)^{1/2}$$

$$\tau = 3.125 \times 10^{-3}$$

where

$$x_i = \bar{m} - \frac{wT^2}{I} i$$

$$\frac{\tau}{\bar{m}} = \frac{3.125 \times 10^{-3}}{8.505 \times 10^{-1}} = 0.367 \times 10^{-2}$$

Hence the percentage probable error of an observation of weight one is 0.37%. The probable error of \bar{m} is

$$\begin{aligned} \tau_{\bar{m}} &= \frac{0.6745}{\sqrt{n-1}} \frac{\sqrt{\left(\sum_{i=1}^n p_i x_i^2 \right)^{1/2}}}{\left(\sum_{i=1}^n p_i \right)^{1/2}} \\ &= \frac{\tau}{\left(\sum_{i=1}^n p_i \right)} = \frac{3.125 \times 10^{-3}}{\sqrt{15.81}} = 7.88 \times 10^{-4} \end{aligned}$$

$$\frac{\tau_{\bar{m}}}{\bar{m}} = \frac{7.88 \times 10^{-4}}{8.505 \times 10^{-1}} = 0.09\%$$

Hence the percentage probable error of \bar{m} is 0.09%.

Assuming that the value of WT^2 is good to at least three significant figures, the percentage probable error in m_i can be used as a percentage probable error in I_{xi} . Since τ affects the third significant figure m_i , values of (WT^2) known to four significant figures are in effect good only to three. Hence we can consider values of (WT^2) that are good either to three or four significant figures as having weight one. The percentage probable error of \bar{m} must be added to the percentage probable error of m_{xi} . Hence a determined value of the moment of inertia is good to $\pm(.37\% + .09\%) = \pm 0.5\%$. (Unless WT^2 is certain only to two significant figures, i. e., the unknown object is quite light.) This error agrees very well with the estimated error.

CONCLUSIONS

The trifilar pendulum is a simple, accurate, and reliable tool for finding moments of inertia. It has these advantages over the torsion rod pendulum: its period is independent of the physical properties of the supporting wires, and the load may be shifted to rotate about an arbitrary axis, if desired. It has no more disadvantages than the torsion rod pendulum, once it is set up.

FUTURE WORK

A four-wire pendulum is possible. An unknown could be balanced right on the rig while a pair of cater-cornered wires are slack. The pendulum would be ready to go as soon as the slack wires were tightened, and a quick-disconnect arrangement can be used to slacken and tighten the wires.

APPENDIX I

DERIVATION OF TRIFILAR PENDULUM EQUATION

Let P be the point from which one of the wires hangs (fig. 14). At rest, its other end is at a point A, right below P. But when the bob turns an angle α , the lower end of the wire moves to point B. Let the line AC be the projection of the arc AB in a plane perpendicular to line AP. The line AC is about equal in length to arc AB.

Now

$$\frac{AC}{L} = \frac{S}{L} \doteq \tan \theta \quad (1)$$

and

$$\frac{AC}{R} \doteq \frac{S}{R} \doteq \tan \alpha \quad (2)$$

if θ and α are kept small

$$S \doteq L\theta \quad (3)$$

and

$$S \doteq R\alpha \quad (4)$$

combining

$$\theta = \frac{R\alpha}{L} \quad (5)$$

Let T_1 be the tension in the wire and w_1 is the part of the total weight pulling down on this wire. Since W_1 and T_1 cancel at rest $W_1 = T_1$.

When the bob is rotated, there is a sideways resultant force,

$$F = -T_1 \sin \theta \quad (6)$$

Again keep θ small, so that $\sin \theta = \theta$ and

$$F = -T_1 \theta \quad (7)$$

This force acts through a moment arm of length R , so there is a restoring torque

$$\tau_1 = -T_1 R \theta \quad (8)$$

In general, $\Sigma \tau = I \ddot{\alpha}$, $\ddot{\alpha}$ the angular acceleration, so that

$$\tau_1 + \tau_2 + \tau_3 = I \ddot{\alpha} \quad (9)$$

Assuming θ the same for all three wires,

$$-R \theta (T_1 + T_2 + T_3) = I \ddot{\alpha} \quad (10)$$

The tensions add up to the total weight without assuming the same tension in each wire. Plugging in this relation and (5),

$$I \ddot{\alpha} + \frac{R^2 \alpha W}{L} = 0 \quad (11)$$

This is an equation for harmonic motion. The frequency is

$$f = \frac{1}{2\pi} \sqrt{\frac{R^2 W}{IL}} \quad (12)$$

The period T is

$$T = 2\pi \sqrt{\frac{IL}{R^2W}} \quad (13)$$

squaring and juggling

$$I = \frac{R^2WT^2}{4\pi^2L} \quad (14)$$

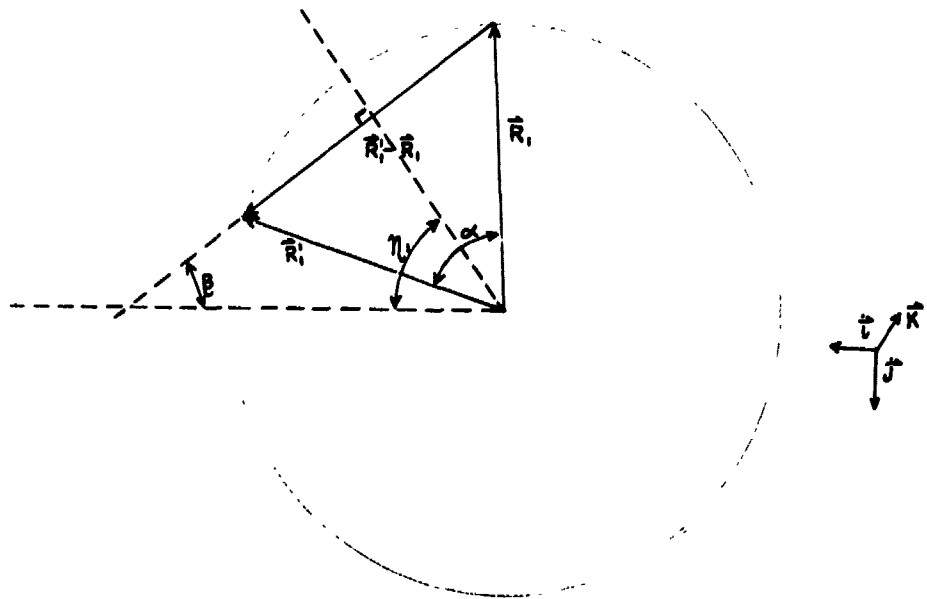
and

$$\frac{I}{WT^2} = \frac{R^2}{4\pi^2L} \quad (15)$$

W is in pounds (weight), R and L in inches and T in seconds; I is in lb. (mass) in.².

APPENDIX II

DISPLACEMENT OF THE LOWER END OF ONE OF THE SUSPENSION WIRES UNDER A COMBINATION OF ROTARY AND SIMPLE PENDULAR MOTIONS



Atop the bob is a circular plate to which the wires are fastened, call it the bottom support plate. Suppose we are looking down at this plate. (See figure above.) Let \vec{R}_1 be the vector from the center of the plate to the fastening point of one of the wires. Let \vec{R}_1 be the vector from the center of the tray to the same point after a rotary shift.

\vec{i} , \vec{j} , and \vec{k} are the rectangular coordinate unit vectors (right-handed system, \vec{k} straight up). If the coordinate system moves sideways with the bob, the shift \vec{S} of the fastening point is just $\vec{R}_1' - \vec{R}_1$. Let η_1 be the angle between the x-axis and the perpendicular bisector of $\vec{R}_1' - \vec{R}_1$. Suppose the coordinates stay fixed with the ground and the bob moves linearly a distance X in the x-direction while it rotates by an angle α . The total shift \vec{S} will now be:

$$\vec{S} = \vec{i}X + (\vec{R}_1' - \vec{R}_1)$$

$(\vec{R}_1' - \vec{R}_1)$ has $\vec{x} + \vec{y}$ components, i.e.

$$(\vec{R}_1' - \vec{R}_1) = (\vec{R}_1' - \vec{R}_1)_x + (\vec{R}_1' - \vec{R}_1)_y$$

but

$$|\vec{R}_1' - \vec{R}_1| = R\alpha$$

so

$$(\vec{R}_1' - \vec{R}_1)_x = \vec{i}R\alpha \cos \beta = \vec{i}R\alpha \sin \eta_1$$

and

$$(\vec{R}_1' - \vec{R}_1)_y = \vec{j}R\alpha \sin \beta = \vec{j}R\alpha \cos \eta_1$$

So

$$\vec{S} = \vec{i}(x + R\alpha \sin \eta_1) + \vec{j}(R\alpha \cos \eta_1)$$

and

$$/S/^2 = (x + R\alpha \sin \eta_1)^2 + (R\alpha \cos \eta_1)^2$$

So

$$|S| = \sqrt{x^2 + (R\alpha)^2} \sqrt{1 + \frac{2R\alpha x \sin \eta_1}{x^2 + (R\alpha)^2}}$$

$/S/$ changes with η and so differs from wire to wire. As a result, the vertical displacement of the suspension point also differs from wire to wire.

APPENDIX III

ERROR IN PERIOD CAUSED BY MISPLACED PHOTOCCELL

The equation of motion for the pendulum is

$$\alpha = \alpha_0 e^{-\gamma t} \cos \omega t \quad (16)$$

α the amplitude, $e^{-\gamma t}$ the damping factor and $\cos \omega t$ the phase factor. Suppose the photocell is triggered when the pendulum reaches angle α_1 , (see diagram #12)

$$\alpha_1 = \alpha_0 e^{-\gamma t_1} \cos \omega t_1 \quad (17)$$

On the second count following the one made at time t_1 , we have again

$$\alpha_1 = \alpha_0 e^{-\gamma t_2} \cos \omega t_2 \quad (18)$$

$$\omega = \frac{2\pi}{T} \quad (19)$$

T the true period and

$$t_2 = t_1 + T + dT \quad (20)$$

dT the difference between measured and true periods. Setting (17) and (18) equal, dividing through, and plugging in (20)

$$e^{-\gamma t_1} \cos \omega t_1 = e^{-\gamma(t_1 + T + dT)} \cos \omega(t_1 + T + dT) \quad (21)$$

Multiplying by $e^{+\gamma(t_1 + T)}$

$$e^{\gamma T} \cos \omega t_1 = e^{-\gamma dT} \cos \omega(t_1 + T + dT) \quad (22)$$

by identity

$$\cos \omega(t_1 + T + dT) = \cos \omega(t_1 + T) \cos \omega dT - \sin \omega(t_1 + T) \sin \omega dT \quad (23)$$

since $dT \ll T$,

$$e^{-\gamma dT} \doteq 1 \quad (24)$$

Putting (23) and (24) into (22),

$$\cos \omega t_1 e^{\gamma T} = \left[\cos \omega(t_1 + T) \cos \omega dT - \sin \omega(t_1 + T) \sin \omega dT \right] \quad (25)$$

by definition

$$\cos \omega(t_1 + T) = \cos \omega t_1 \quad (26)$$

and

$$\sin \omega(t_1 + T) = \sin \omega t_1 \quad (27)$$

again since $dT \ll T$

$$\cos \omega dT \approx 1 \quad (28)$$

and

$$\sin \omega dT = \omega dT \quad (29)$$

Putting (19), (26), (27), (28) and (29) into (25) and rearranging

$$\frac{-\cos \omega t_1 \left[e^{\gamma T} - 1 \right]}{2\pi \sin \omega t_1} = \frac{dT}{T} \quad (30)$$

Putting in a phase angle ϕ such that

$$\phi = \pi/2 - \omega t_1 \quad (31)$$

we obtain

$$\frac{dT}{T} = \frac{\tan \phi}{2\pi} \left[e^{\gamma T} - 1 \right] \quad (32)$$

but

$$\alpha_1 = \alpha_0 e^{-\gamma t_1} \sin \phi \quad (33)$$

so that

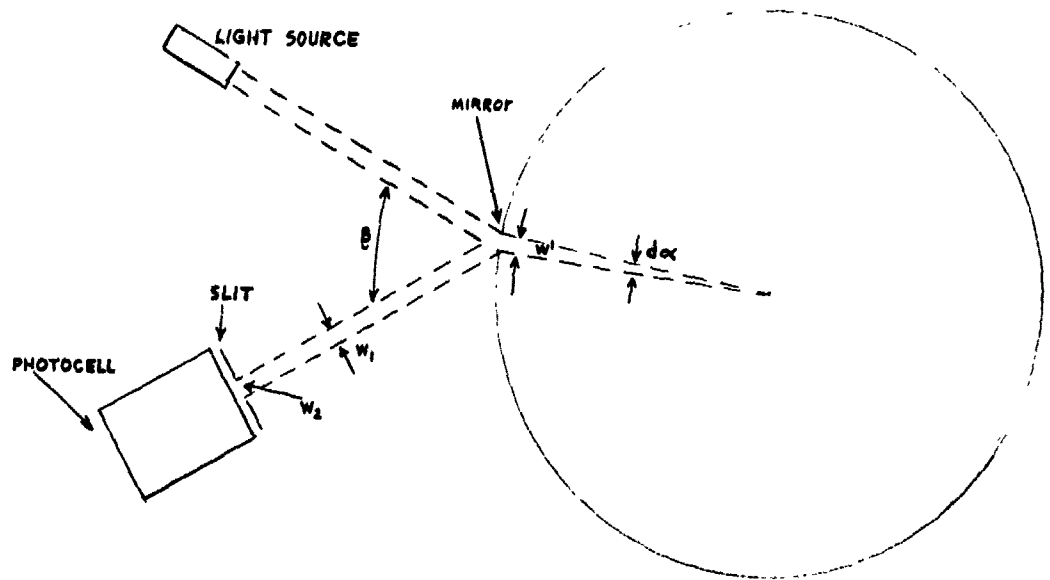
$$\tan \phi = \frac{\alpha_1}{\sqrt{\left(\alpha_0 e^{-\gamma t_1}\right)^2 - \alpha_1^2}} \quad (34)$$

and

$$\frac{dT}{T} = \frac{1}{2\pi} \frac{\alpha_1}{\sqrt{\left(\alpha_0 e^{-\gamma t_1}\right)^2 - \alpha_1^2}} \left[e^{\gamma T} - 1 \right] \quad (35)$$

APPENDIX IV

EFFECT ON MIRROR AND PHOTOCELL WIDTHS ON MEASURED PERIOD OF THREE-WIRE PENDULUM



The bob is at its equilibrium position, so light reflects into the slit. Assume parallel rays hit the mirror, so the beam width won't change from the mirror to the slit. The width of the reflected beam will be

$$w_1 = w' \cos \beta/2 \quad (36)$$

or at worst

$$w_1 = w' \quad (37)$$

where w' is the mirror width and β is the angle between incident and reflected beams. w_2 is the width of the slit in front of the photocell. The angle β can vary by as much as

$$d\beta = \frac{w_1 + w_2}{D} \quad (38)$$

from period to period at the moment the photocell fires.

The variation in the angle α is approximately

$$d\alpha = \frac{d\beta}{2} \quad (39)$$

that is

$$d\alpha = \frac{w_1 + w_2}{2D} \quad (40)$$

The uncertainty in the angle α leads to a possible error in the exact time that the photocell is fired. This possible error is at most the time required to sweep out an angle $d\alpha$.

Since

$$\frac{d\alpha}{\Delta T} = \dot{\alpha} \quad (41)$$

$$\Delta T = \frac{d\alpha}{\dot{\alpha}} \quad (42)$$

but

$$\alpha = \alpha_0 \sin \omega t \quad (43)$$

so

$$\dot{\alpha} = \omega \alpha_0 \cos \omega t \quad (44)$$

plugging (19) and (44) into (42)

$$\Delta T = \frac{T d\alpha}{2\pi \alpha_0 \cos \omega t} \quad (45)$$

since α is about 0,

$$\Delta T = \frac{T}{2\pi} \frac{d\alpha}{\alpha_0} \quad (46)$$

Ten periods are timed, and the starting and ending times may be off by this much. So

$$\frac{dT_3}{T} = \frac{2\Delta T}{10T} \quad (47)$$

Putting (46) and (40) in (47)

$$\frac{dT}{T} = \frac{(w_1 + w_2)}{20\pi D \alpha_0} \quad (48)$$

It is easy to show that if the light source has a finite width w_3 in the plane of motion and looks at the mirror from a distance D (and not from infinity),

$$\frac{dT}{T} = \frac{1}{20\pi D \alpha_0} (2w_1 + w_2 + w_3) \quad (49)$$

DATA TABLE

Standard No.	Orientation	W _{s+o} lb(wt)	T _{s+o} sec	W _{s+o} T _{s+o} ² lb(wt)sec ²	W _{s+o} T _{s+o} ² - W _o T _o ² lb(wt)sec ²	I _{s+o} - I _o lb(mass)in ²
Holder only						
1	Horizontal	0.375	2.449	2.249	0.000	0.000
2	"	0.877	1.676	2.463	0.214	0.240
3	"	1.379	1.446	2.883	0.634	0.7389
4	"	1.882	1.411	3.747	1.498	1.761
5	"	2.384	1.489	5.285	3.036	3.565
6	"	2.886	1.632	7.685	5.436	6.409
A	"	3.387	1.819	11.208	8.959	10.557
B	"	0.878	1.899	3.17	0.917	1.09
C	"	1.387	1.735	4.18	1.93	2.27
D	"	1.885	1.666	5.23	2.98	3.51
E	"	2.384	1.630	6.334	4.085	4.824
F	"	2.889	1.615	7.534	5.285	6.240
	"	3.393	1.609	8.784	6.535	7.725
Vertical						
1		0.877	1.718	2.588	0.339	0.392
2	"	1.379	1.454	2.915	0.666	0.783
3	"	1.882	1.313	3.244	0.995	1.177
4	"	2.384	1.228	1.570	1.346	1.570
5	"	2.886	1.165	1.967	1.667	1.967
6	"	3.387	1.119	4.241	1.992	2.349
A	"	0.878	1.627	2.324	0.075	0.080
B	"	1.387	1.350	2.528	0.279	0.322
C	"	1.885	1.234	2.870	0.621	0.719
D	"	2.384	1.184	3.324	1.093	1.278
E	"	2.889	1.170	3.955	1.706	2.002
F	"	3.393	1.177	4.699	2.450	2.878
Unknown ^a	"	2.826	1.338	5.059	2.810	3.304**
"	Horiz. hub at 0°	2.826	1.565	6.922	4.673	5.494**
"	" " 90°	2.826	1.602	7.253	5.004	5.884**

^aA mechanical time fuze

** Experimentally Determined Values

APPENDIX V

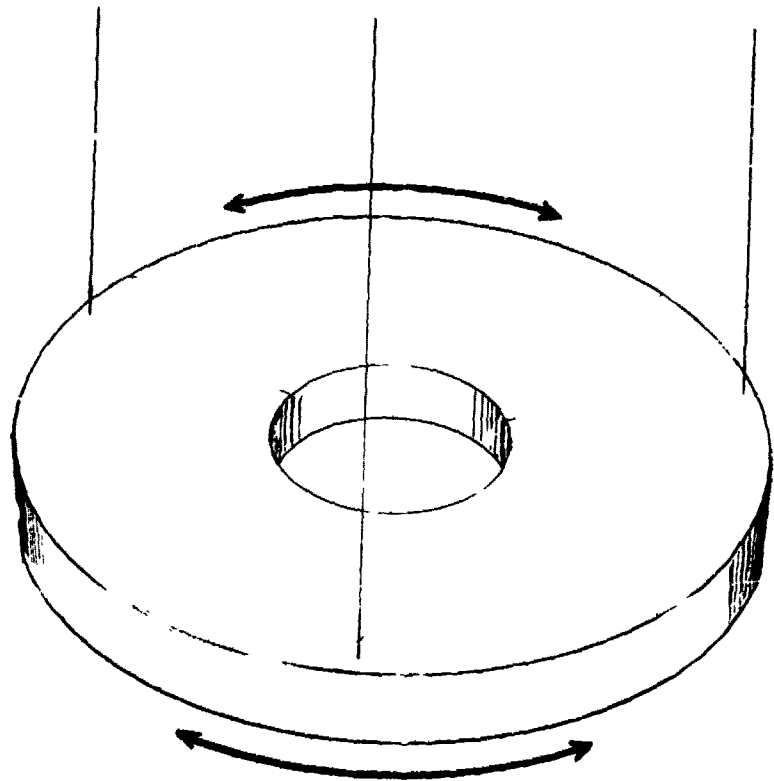


Figure 1. Trifilar Pendulum, Schematic Diagram

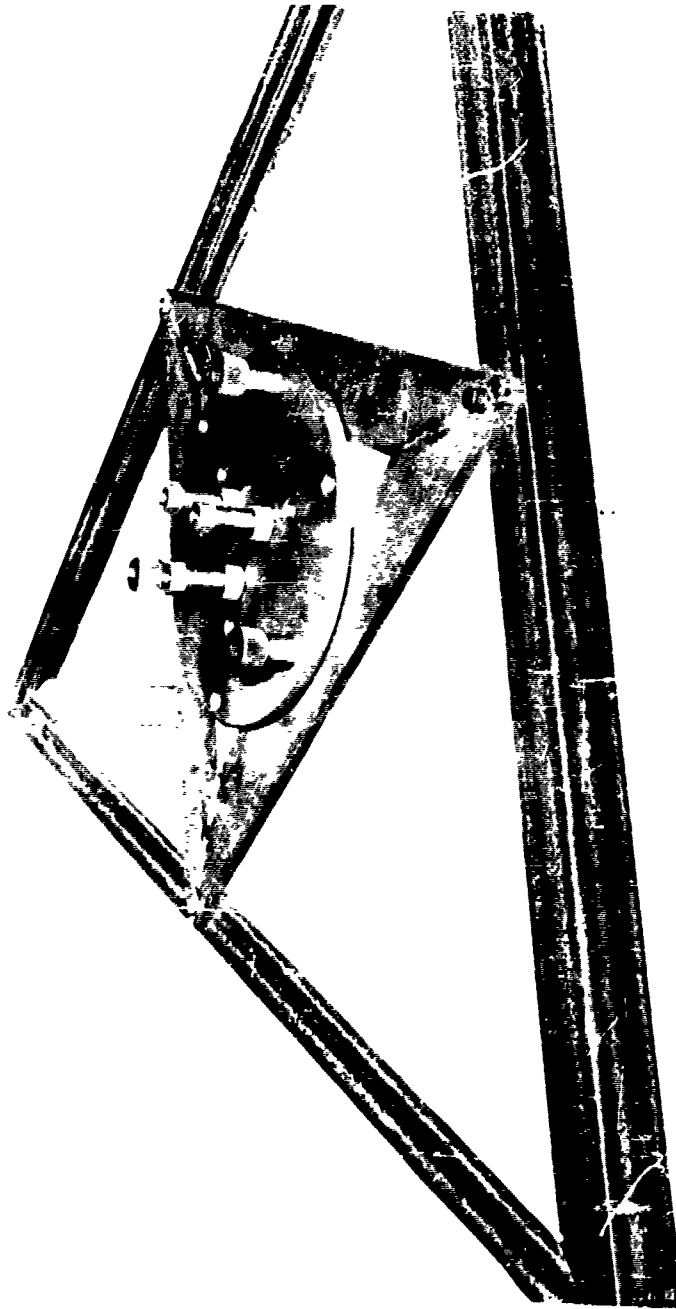


Figure 2. Upper Support Assembly of Trifilar Pendulum, Looking Down

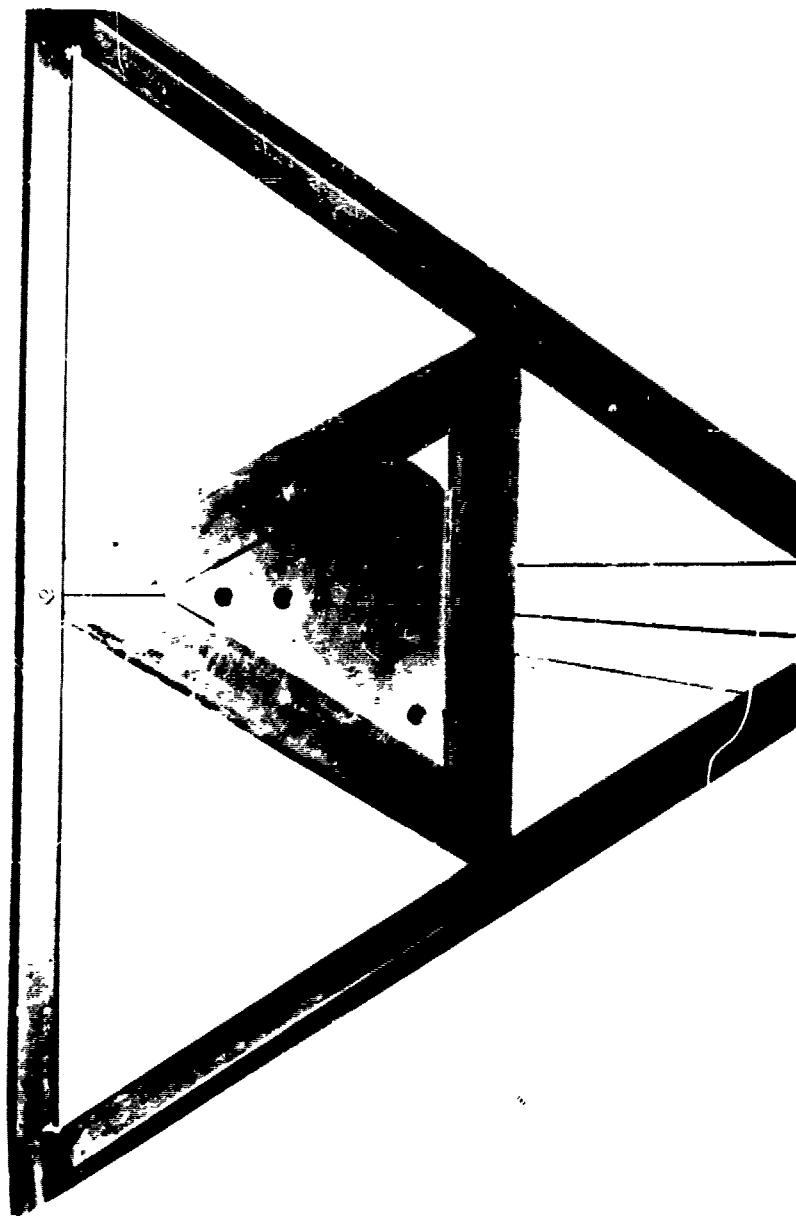


Figure 3. Upper Support Assembly of Trifilar Pendulum, Looking Up

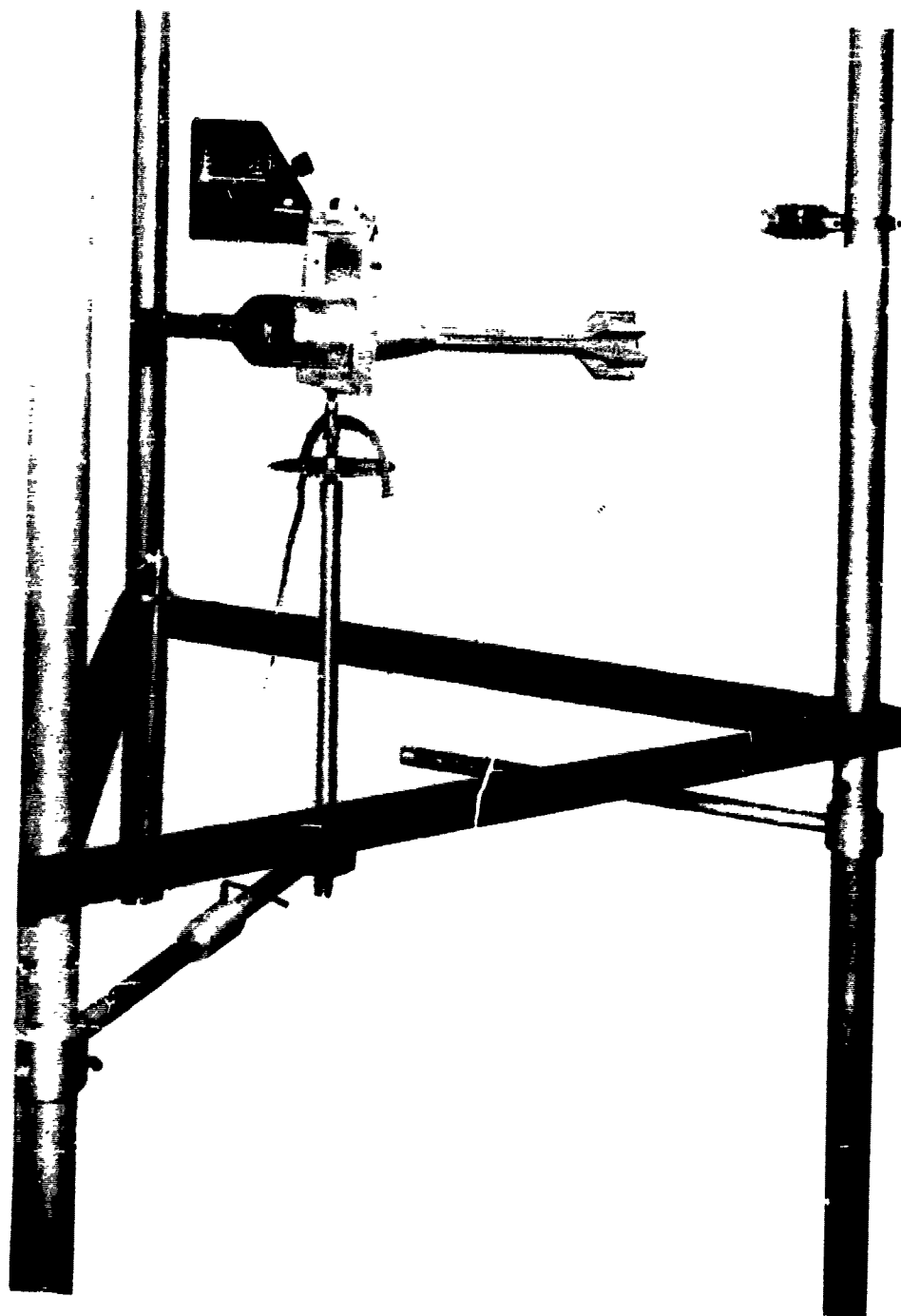


Figure 4. Original Design of Three-Wire Pendulum Frame

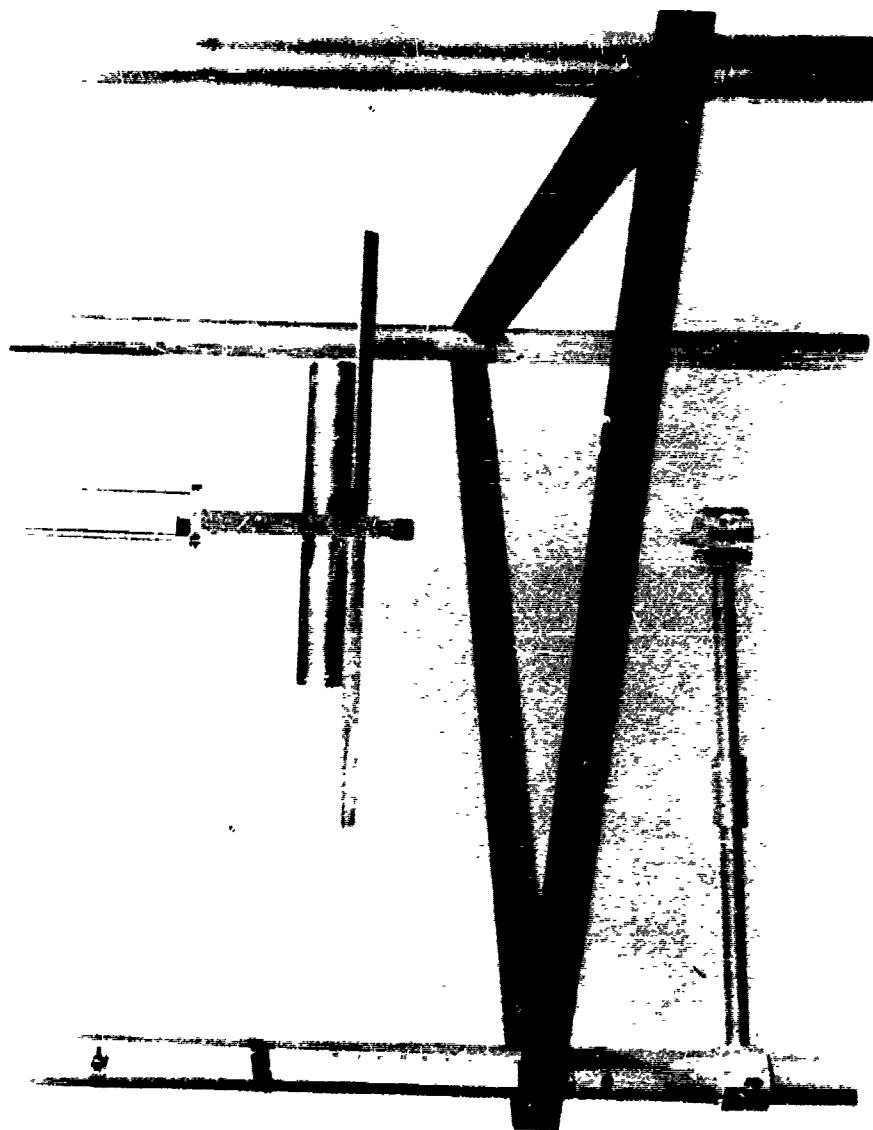


Figure 5. Holding Frame of Trifilar Pendulum with Test Object,
Side View, and Guidance Setup

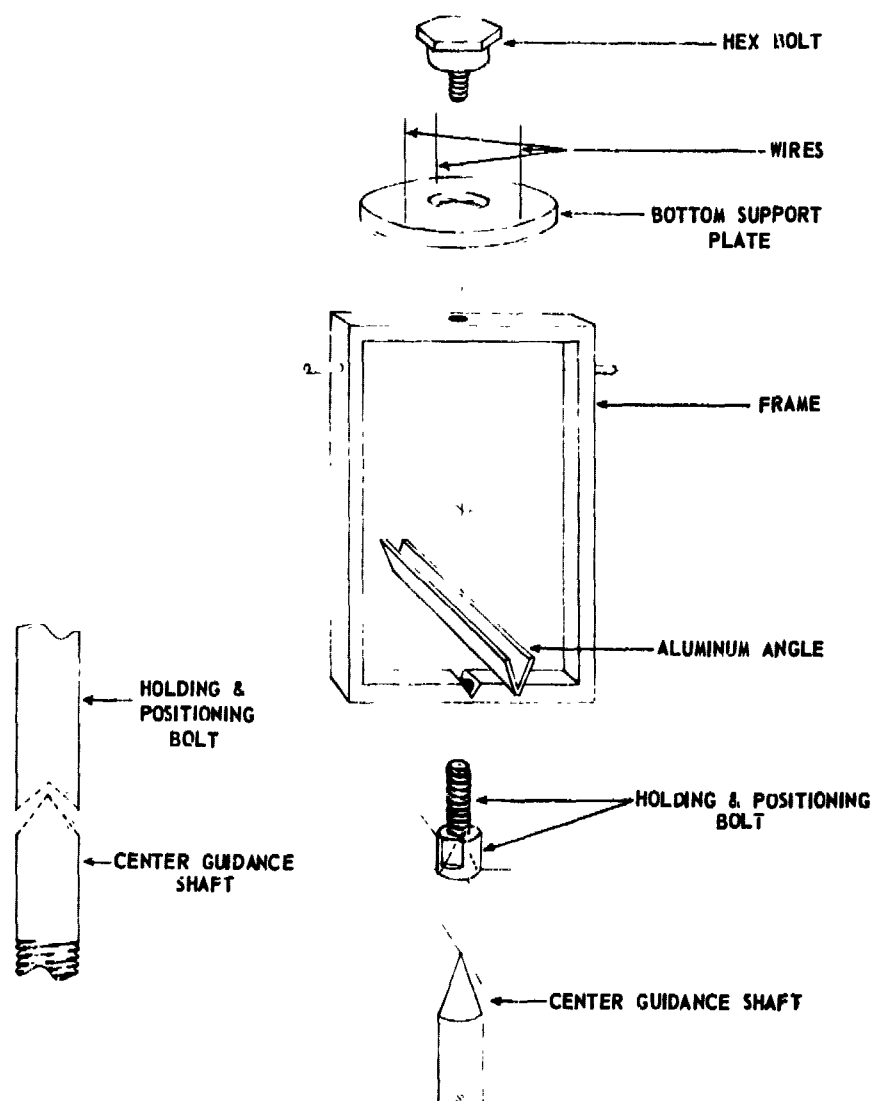


Figure 6. Aluminum Holding Frame of Trifilar Pendulum, Exploded View
 INSET: Point Contact Between Bored Cylinder and Guidance Plug



Figure 7. Expansion Plug for Hanging Objects Having Axial Mounting
Holes on Three-Wire Pendulum

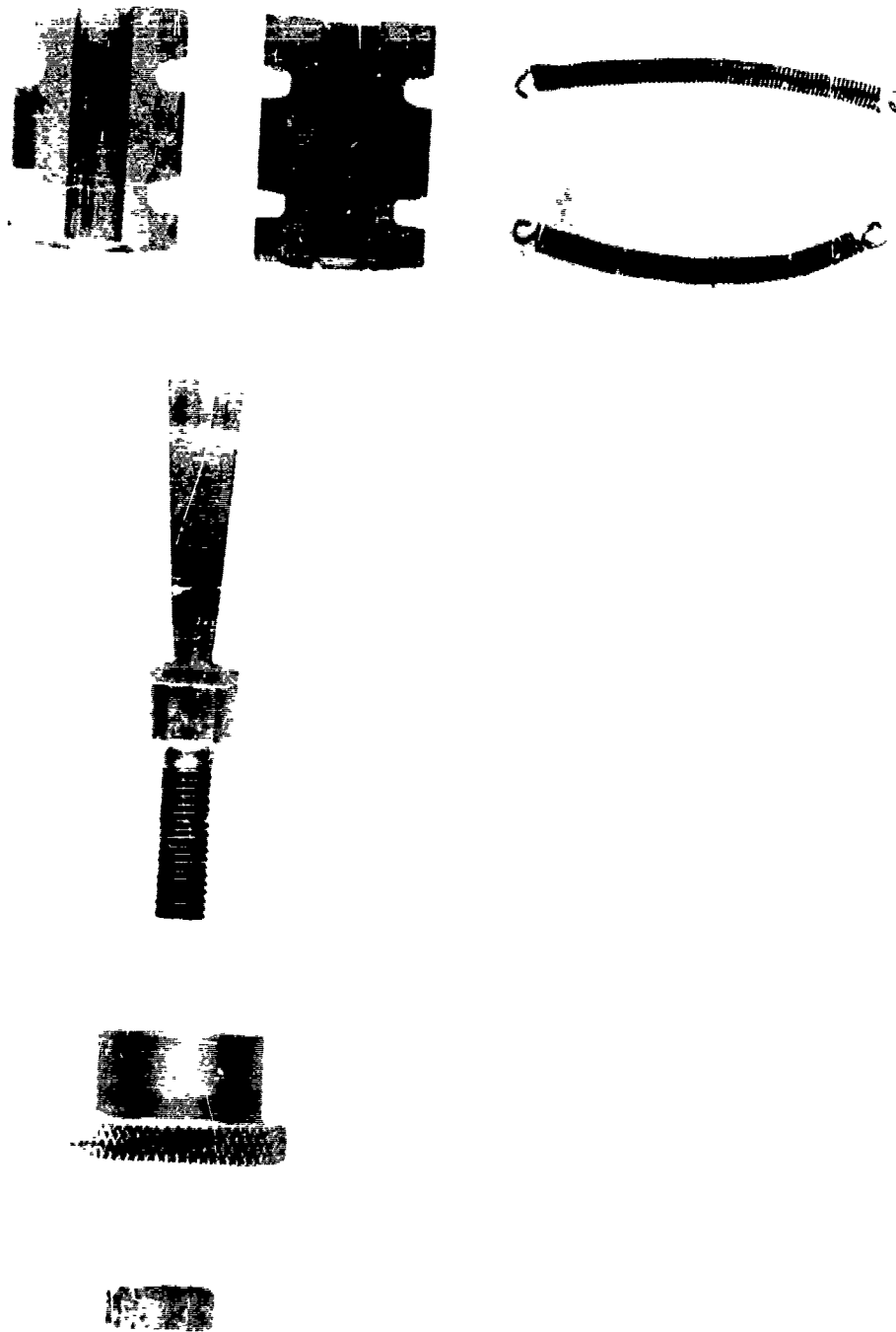


Figure 8. Expansion Plug for Hanging Objects Having Axial Mounting Holes on Trifilar Pendulum, Taken Apart



Figure 9. Spacer for Measuring Length of Suspension Wires of Trifilar Pendulum

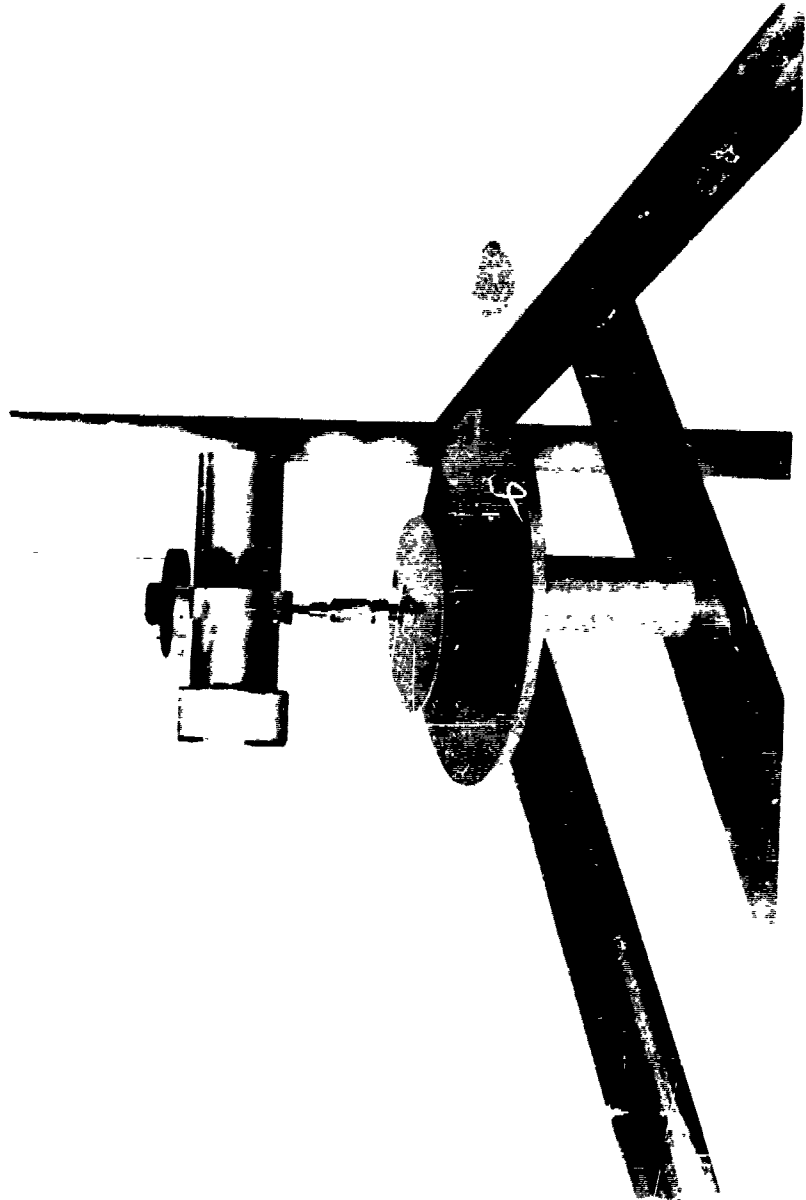
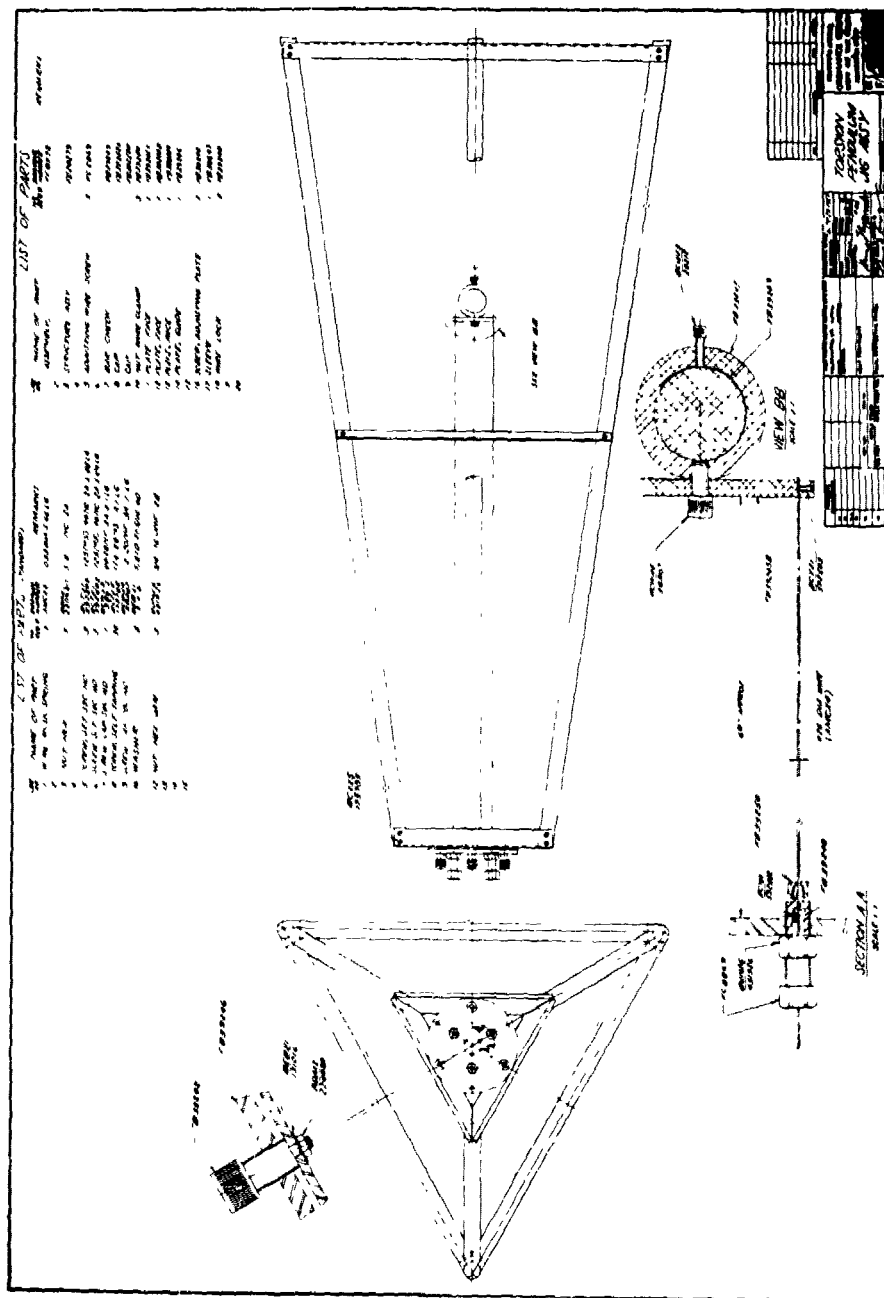


Figure 10. Setup for Fixing Amplitude and Starting Without Tremor



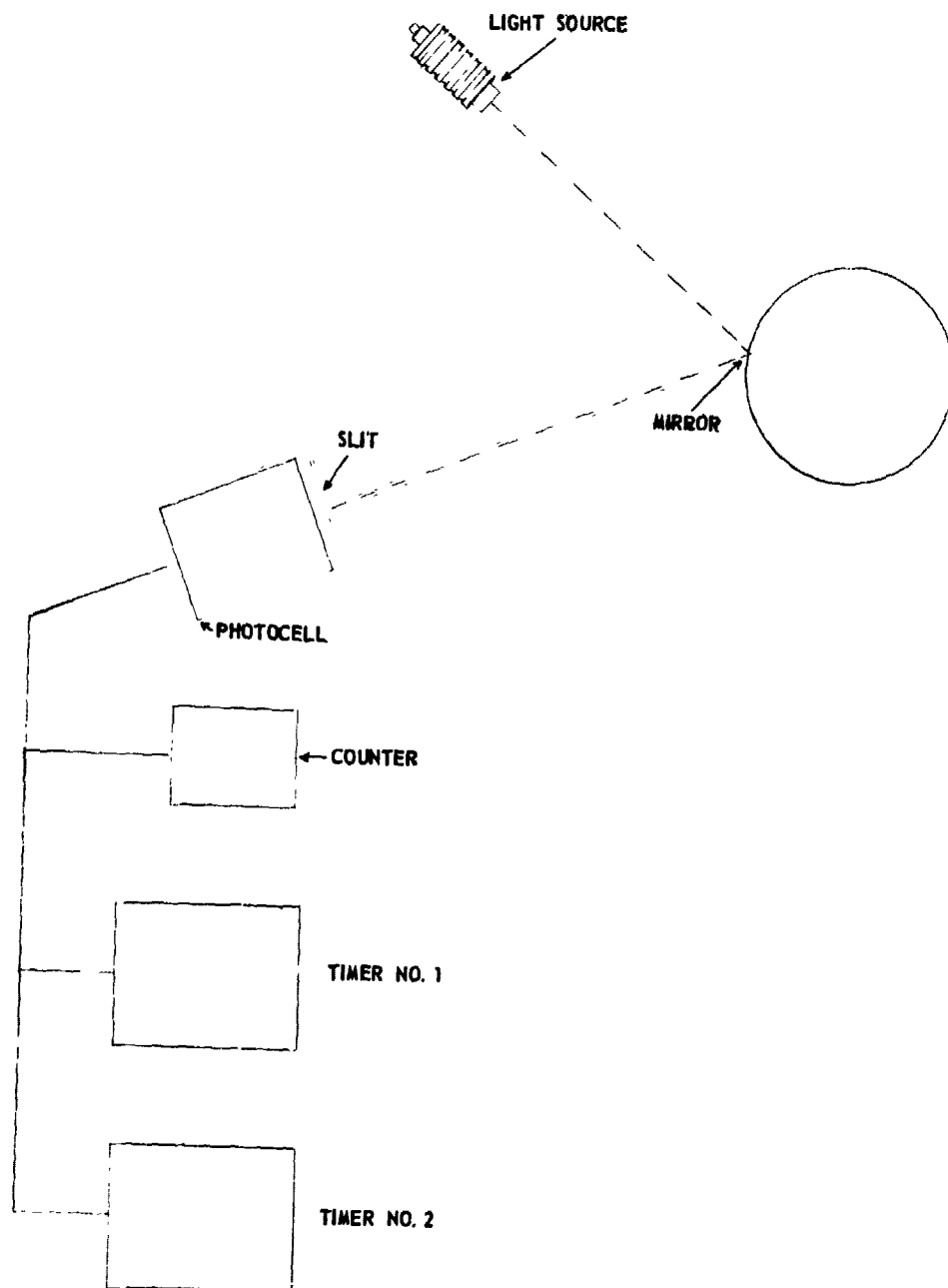


Figure 12. Timing Setup for Three-Wire Pendulum

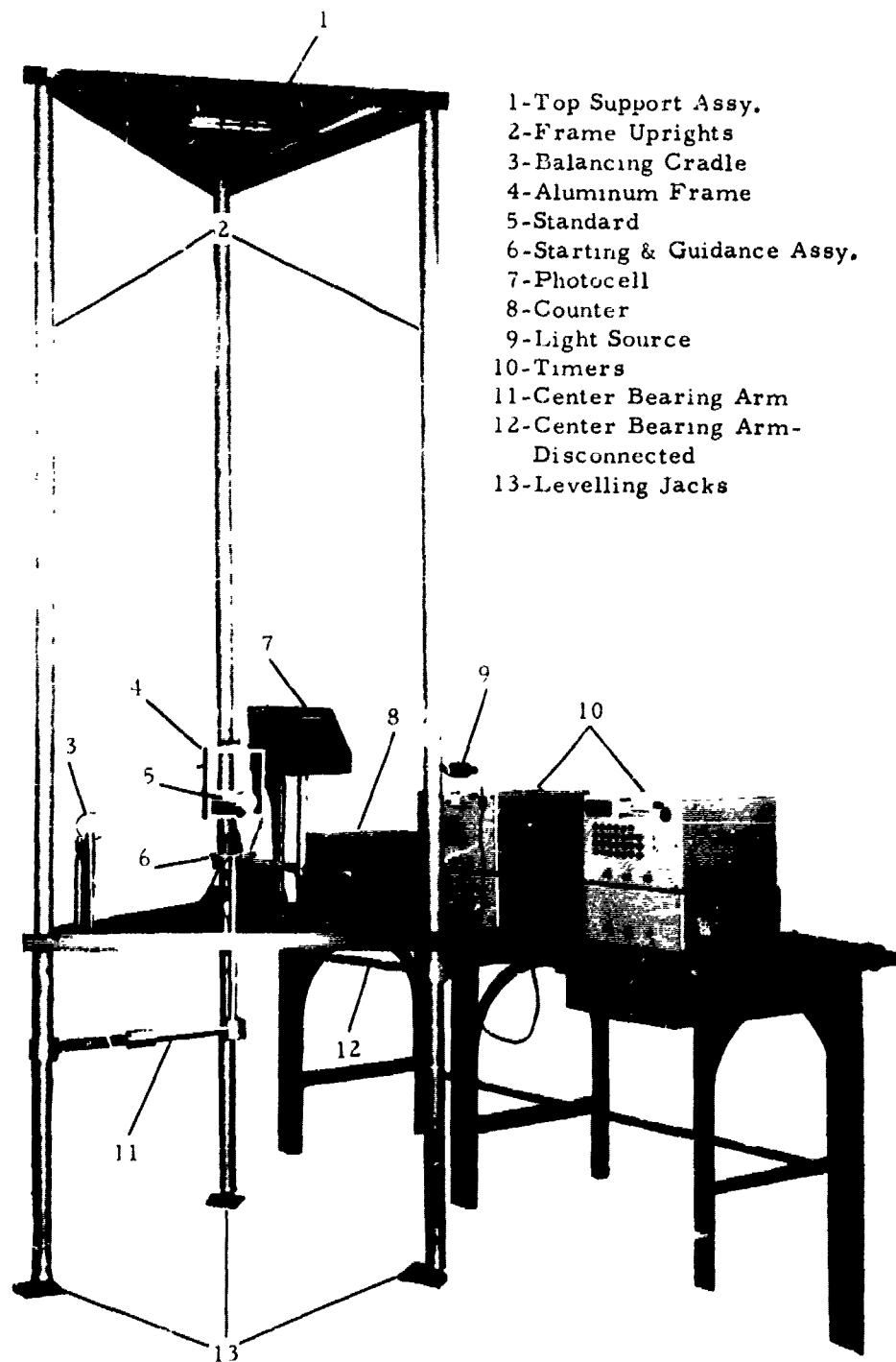
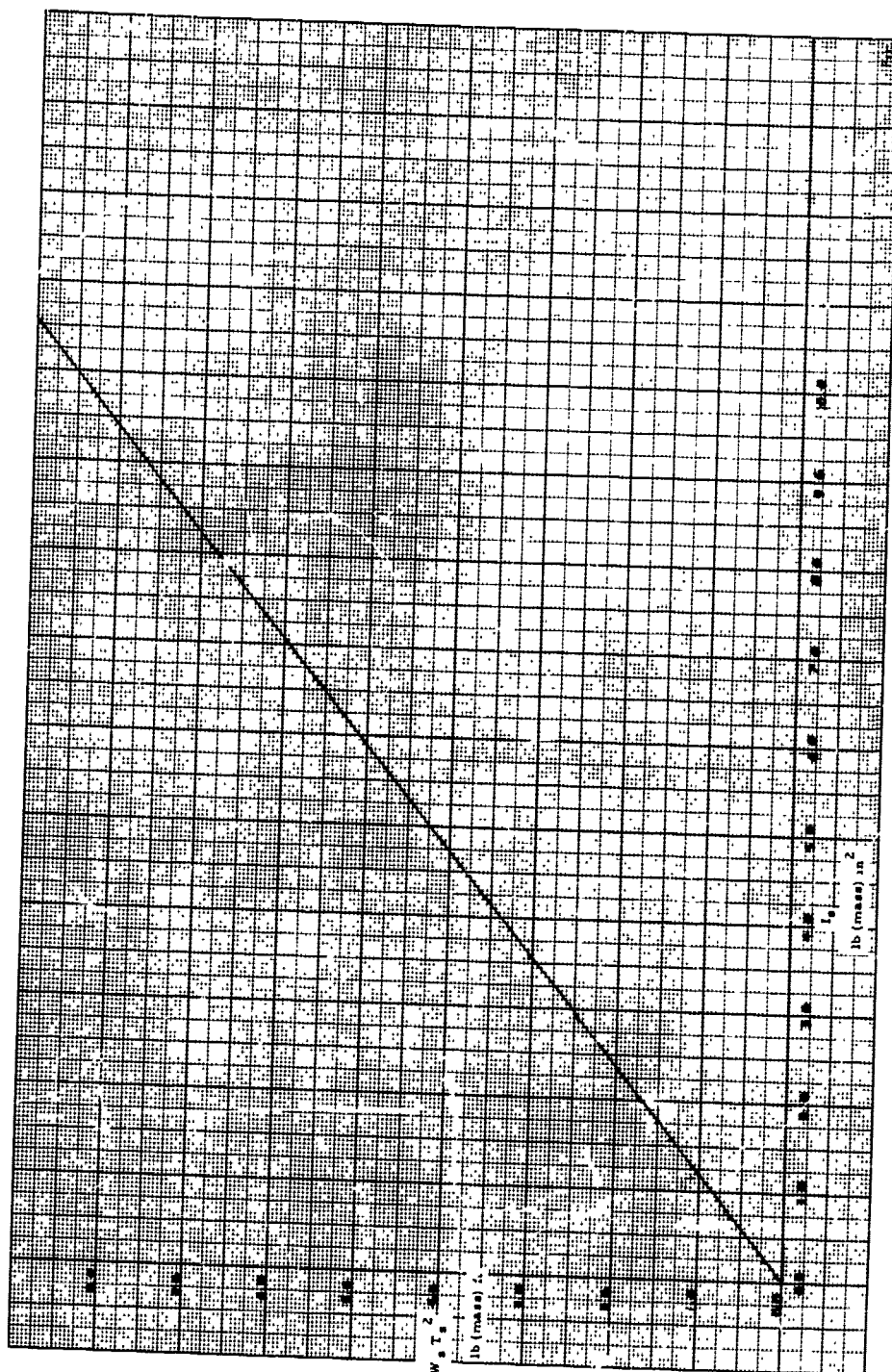
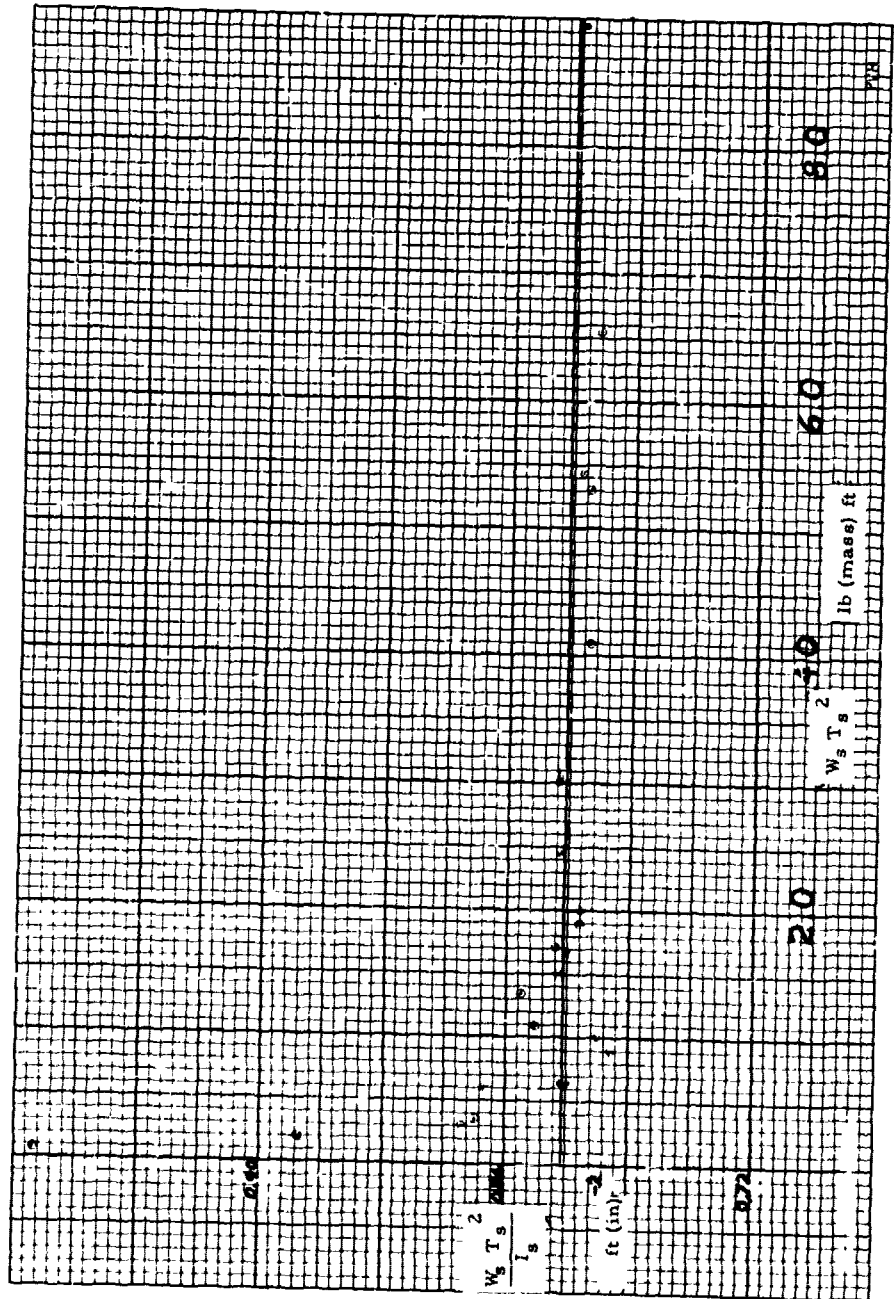


Figure 13. Entire Trifilar Pendulum Setup



Graph #1



Graph #2

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There are specialized techniques used in operating the pendulum and some unique features in its design for which patents have been applied. The need for such unique features is established, and error analysis shows this trifilar pendulum accurate to 0.5%.

It is concluded that the trifilar pendulum, if properly constructed, has certain advantages over the torsion rod pendulum with no more disadvantages.

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